SELF-SYNCHRONIZATION AND SELF-STABILIZATION OF WALKING GAITS MODELED BY THE 3D LINEAR INVERTED PENDULUM MODEL

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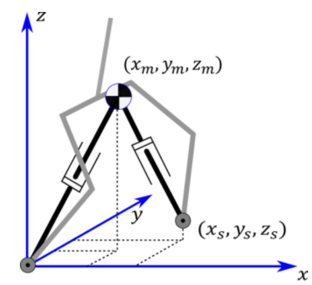
LIP Model

- Three assumptions:
  - 1. punctual mass
  - 2. massless beam
  - 3. altitude is constant



- In order to explore simultaneously the self-synchronization and self-stabilization of periodic orbits for many step length and width, a dimensionless dynamic model of the pendulum will be used

$$(X, Y, z_m, X_s, Y_s, z_s)=(\frac{x_m}{s}, \frac{y_m}{D}, z_m, \frac{x_s}{S}, \frac{y_s}{D}, z_s)$$

$$\begin{cases} x_m = P_x + \dfrac{z_m \ddot{x}_m}{g} \\ y_m = P_y + \dfrac{z_m \ddot{y}_m}{g} \end{cases} \xrightarrow[\text{Normalized variables}]{\text{Point foot}} \begin{cases} \ddot{X} = \omega^2 X \\ \ddot{Y} = \omega^2 Y \end{cases}$$

where ω is $\sqrt{\frac{g}{z_m}}$

Notation introduction

- State:
  - $X_{k+1}^{+} = X_k^{-} - X_{s,k}^{-}$
  - $Y_{k+1}^{+} = -Y_k^{-} + Y_{s,k}^{-}$
  - $X_k^{+} = X^{*+} + \delta X_k^{+}$
  - $Y_k^{+} = Y^{*+} + \delta Y_k^{+}$
  - $X_k^{-} = X^{*-} + \delta X_k^{-}$
  - $Y_k^{-} = Y^{*-} + \delta Y_k^{-}$
  - $\dot{X}_{k+1}^{+} = \dot{X}_k^{-}$
  - $\dot{Y}_{k+1}^{+} = -\dot{Y}_k^{-}$
  - $\dot{X}_k^{+} = \dot{X}^{*+} + \delta \dot{X}_k^{+}$
  - $\dot{Y}_k^{+} = \dot{Y}^{*+} + \delta \dot{Y}_k^{+}$
  - $\dot{X}_k^{-} = \dot{X}^{*-} + \delta \dot{X}_k^{-}$
  - $\dot{Y}_k^{-} = \dot{Y}^{*-} + \delta \dot{Y}_k^{-}$
- The sign (-) denotes the state before impact (at the end of a step) and (+) denotes the state after impact (at the beginning of a step)

Synchronization measure

- The synchronization measure

$$L = \dot{X}\dot{Y} - \omega^2 XY$$

and the orbital energy

$$E_x = \dot{X}^2 - \omega^2 X^2$$
$$E_y = \dot{Y}^2 - \omega^2 Y^2$$

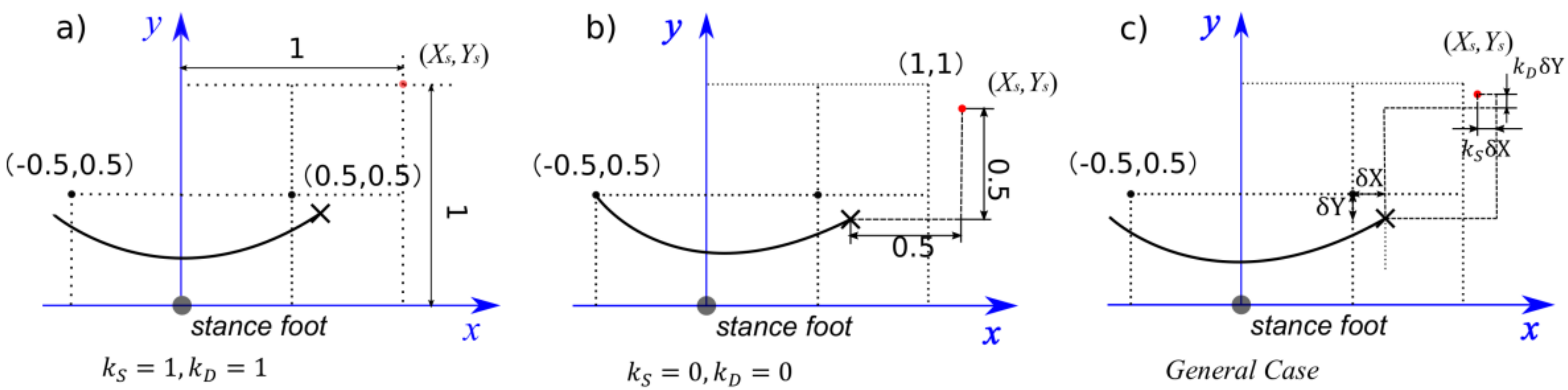
are conserved during a single support phase

- We can say that the solution in one step is synchronized if and only if the synchronization measure is zero.

Swing foot motion

- For general case, the pose of the swing foot at the end of the step is expressed as:

$$
\begin{aligned}
X^{-}_{s,k} &= (1-k_S)(X^{-}_{k} - X^{*-}) + 1 \\
Y^{-}_{s,k} &= (1-k_D)(Y^{-}_{k} - Y^{*-}) + 1
\end{aligned}
$$



a) the stride is imposed (1,1); b) error in position of the CoM is nullified; c) the general case

Phasing variable

- Definition:

A normalized variable monotonically increasing from 0 to 1 during one step

- Advantage:

To describe the desired trajectory of the controlled variables and to ensure the joint coordination.

$$X_s = X_s(\Phi),\ Y_s = Y_s(\Phi),\ z_s = z_s(\Phi)$$

Condition of transition based on time

- In many literatures, researchers define the change of support as function of a reference duration of a step T^*.

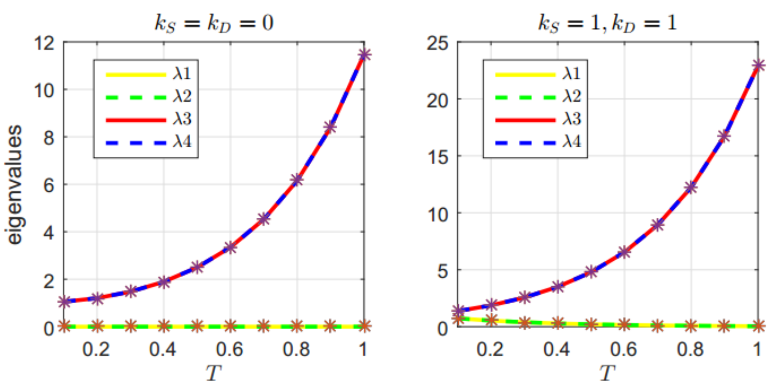
- With this method, the phasing variable is defined as:

$$\Phi = \frac{t}{T^*}$$

➢ However, this method has been proven to be unstable is this paper (will be explained later).

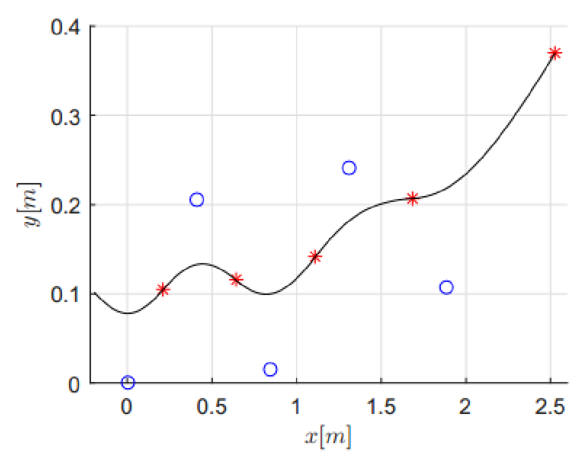
Condition of transition based on time

- Poincaré return map is used to analyze the stability



Jacobian of the Poincaré return map at the fixed point is calculated numerically in the coordinate system $(X_k^-, Y_k^-, L_k^-, K_k^-)$, where L_k^- is the synchronization measure at the end of step k and K_k^- is the kinetic energy at the end of the step k.

Simulation:
$k_S = k_D = 0$, S=0.4m, D=0.2m, z_m=1m, T=0.6s



Condition of transition based on the position of the CoM

- The robot switches its stance leg when the CoM crosses the switching manifold:

$$\mathbf{S} = \{(X, Y) | (X - X^{*-}) + C(Y - Y^{*-}) = 0\}.$$

- which is presented by the red line in the figure below
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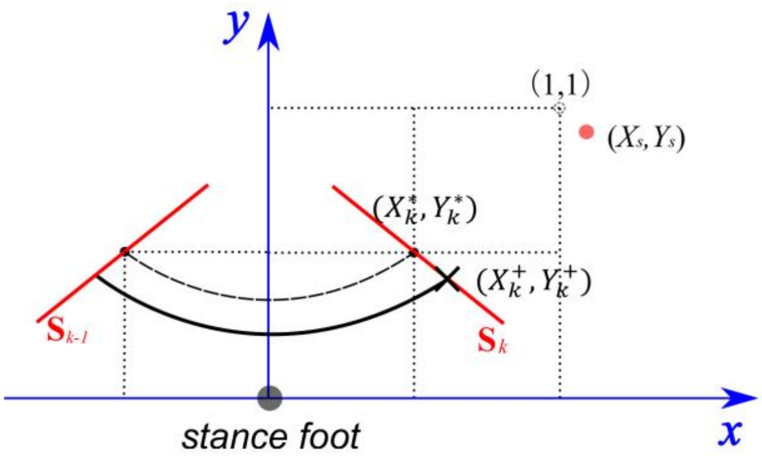


Phasing variable for self-synchronization

- Define the phasing variable:

$$\Phi = aX + bY + cXY + dX^2 + eY^2 + f$$

Knowing that:

$$\Phi(X^+, Y^+) = 0$$

$$\Phi(X^-, Y^-) = 1,\ X^- = X^{*-} - C\delta, Y^- = Y^{*-} + \delta$$

$$\Rightarrow \begin{cases} aX^+ + bY^+ + cX^+Y^+ + d(X^+)^2 + e(Y^+)^2 + f = 0 \\ aX^{*-} + bY^{*-} + cX^{*-}Y^{*-} + d(X^{*-})^2 + e(Y^{*-})^2 + f = 1 \\ -cC + dC^2 + e = 0 \\ -aC + b - cCY^{*-} + cX^{*-} - 2dCX^{*-} + 2eY^{*-} = 0. \end{cases}$$

Phasing variable for synchronization

- There are six variables while the number of equations is only four. Take c=0 in order to reduce the number of terms. Then we get:

$$a = -d((X^-)^* + X^+ + C(Y^-)^*) + dCY^+ + \frac{1}{(X^-)^* - X^+ + C((Y^-)^* - Y^+)}$$

$$b = dC((X^-)^* - X^+ + C((Y^-)^* + Y^+)) + \frac{C}{(X^-)^* - X^+ + C((Y^-)^* - Y^+)}$$

$$e = -dC^2$$

$$f = d((X^-)^* + C(Y^-)^*)(X^+ - CY^+) + \frac{X^+ + CY^+}{-(X^-)^* + X^+ - C(Y^-)^* + CY^+}$$

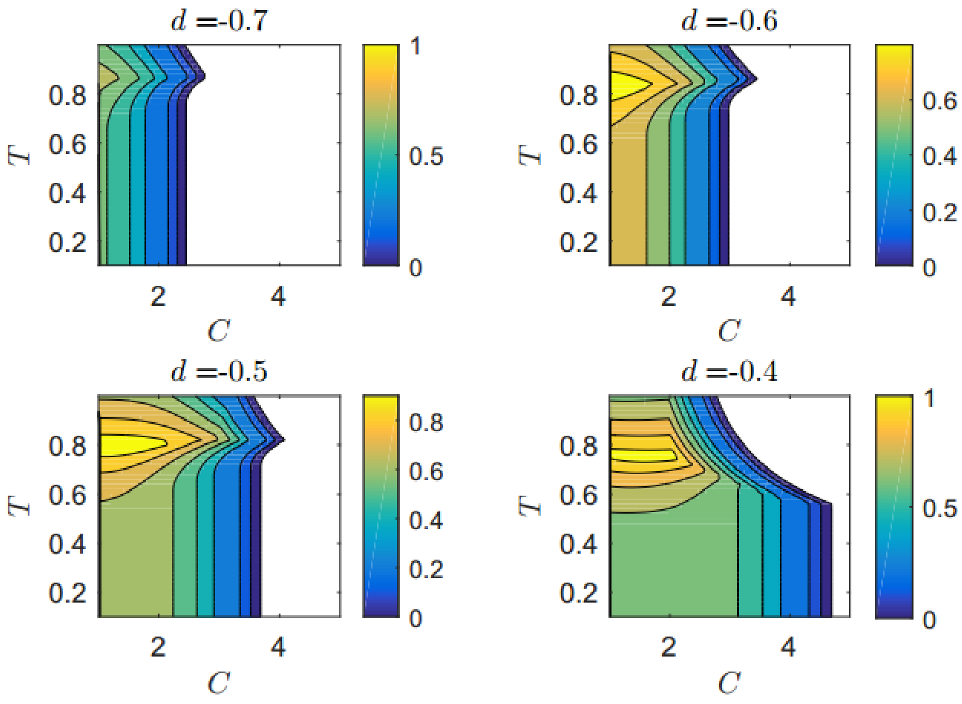
- Rearrange the equations:

$$\Phi = \frac{M_1}{M_2} + dM_3M_4$$

$$\begin{aligned} M_1 &= X - X^+ + CY - CY^+ \\ M_2 &= X^{*-} - X^+ + CY^{*-} - CY^+ \\ M_3 &= X - X^{*-} + CY - CY^{*-} \\ M_4 &= X - X^+ - CY + CY^+. \end{aligned}$$

Phasing variable for synchronization

- The choice of d will determine if the phasing variable will be monotonic



Stability study for general case

- Since X and Y are coupled, only three varibles (X_k^-, L_k^-, K_k^-) are taken as the coordinate system of Poincaré return map. The expression of the Jacobian of Poincaré return map in analytical form can be deduced.

$$J = \begin{bmatrix} -\frac{k_D+\alpha C k_S}{(k_S+\alpha C)} & \frac{4\alpha C k_S}{\omega^2(1+\alpha C)(\alpha-1)} & 0 \\ J_{21} & \frac{2\alpha(k_D-Ck_S)+(1-\alpha C)(\alpha+1)}{(1+\alpha C)(\alpha-1)} & 0 \\ * & * & 1 \end{bmatrix}$$

- where
- $$J_{21} = \frac{(k_D+\alpha C k_S)(C-1+Ck_S-k_D)\omega^2}{2(1+\alpha C)Ck_S}$$

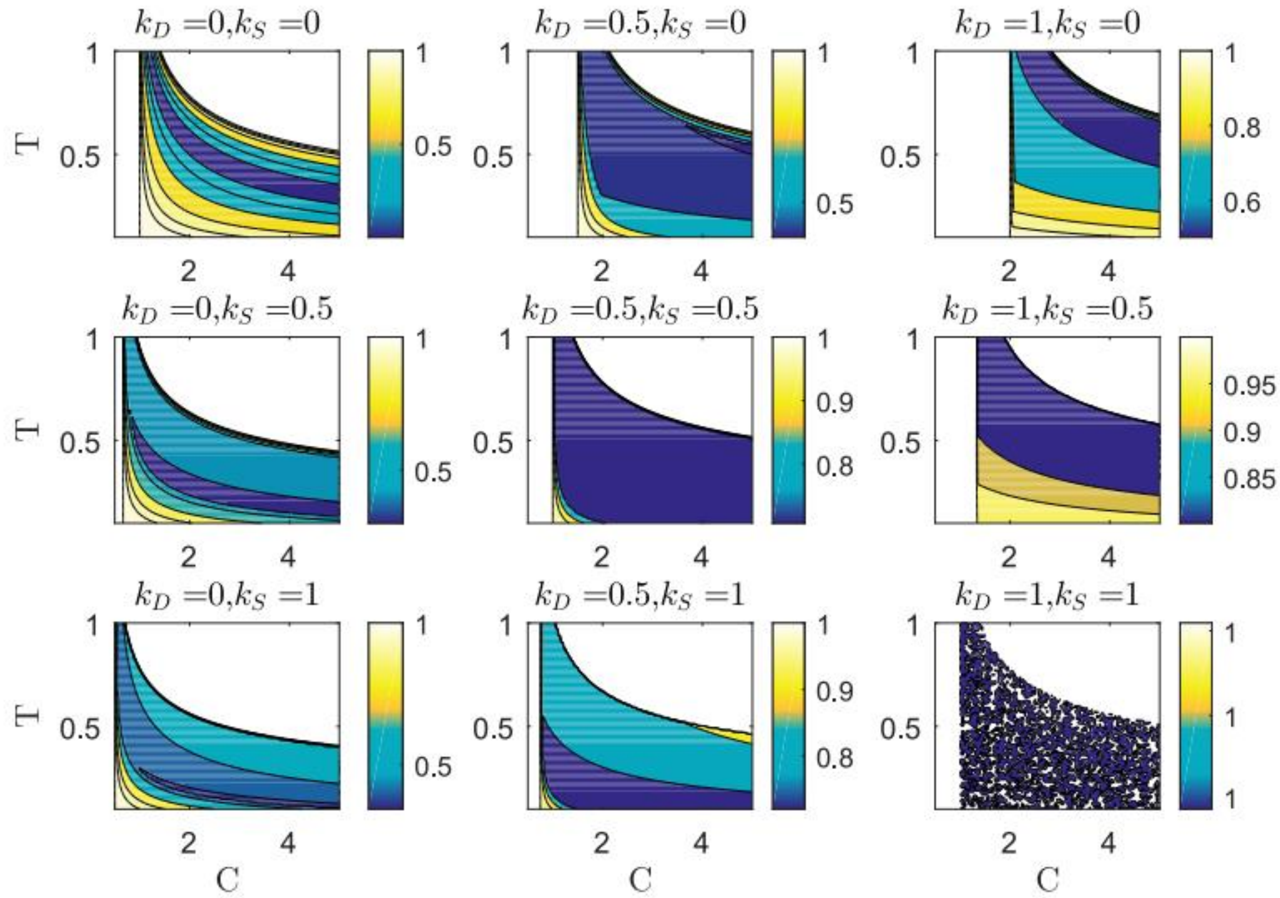
The eigenvalue associated with K_k^- is always 1, and this means that the walking velocity cannot be controlled.

Change of support as function of the pose of the CoM

	parameters	Jacobian matrix
Particular case 1	$k_S = k_D = 0$	$J = \begin{bmatrix} 0 & 0 & 0 \\ \frac{\alpha(-1+C)\omega^2}{2(1+\alpha C)} & \frac{(-1+\alpha C)(\alpha+1)}{(1+\alpha C)(1-\alpha)} & 0 \\ * & * & 1 \end{bmatrix}$
Particular case 2	$k_D=1$, $k_S<1$	$J = \begin{bmatrix} -\frac{1+\alpha C k_S}{1+\alpha C} & \frac{4\alpha C k_S}{\omega^2(1+\alpha C)(\alpha-1)} & 0 \\ \frac{(-2+C+Ck_S)(1+\alpha C k_S)\omega^2}{2C(1+\alpha C)k_S} & \frac{(-1+\alpha C)(\alpha+1)-2\alpha(1-Ck_s)}{(1+\alpha C)(\alpha-1)} & 0 \\ * & * & 1 \end{bmatrix}$
Particular case 3	$k_D=k_S=1$	$J = \begin{bmatrix} -1 & \frac{4\alpha C}{\omega^2(1+\alpha C)(\alpha-1)} & 0 \\ \frac{(-1+C)\omega^2}{C} & \frac{2\alpha(1-C)+(1-\alpha C)(\alpha+1)}{(1+\alpha C)(\alpha-1)} & 0 \\ * & * & 1 \end{bmatrix}$

Note: it can be proven that for the case 3, there is always more than one eigenvalues with a norm greater than 1

Eigenvalues for different kD and kS



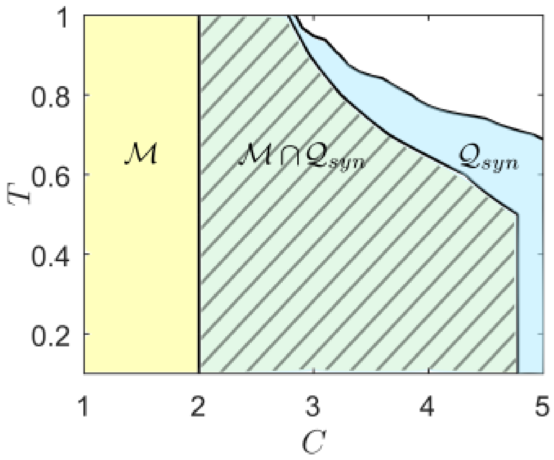
Choice of C

- The manifold of *C* and *T* that makes the walking gait self-synchronized is defined:

$$\mathcal{Q}_{syn} := \{(C,T)|\lambda_{1,2}(C,T) < 1, \lambda_3(C,T) = 1\}$$

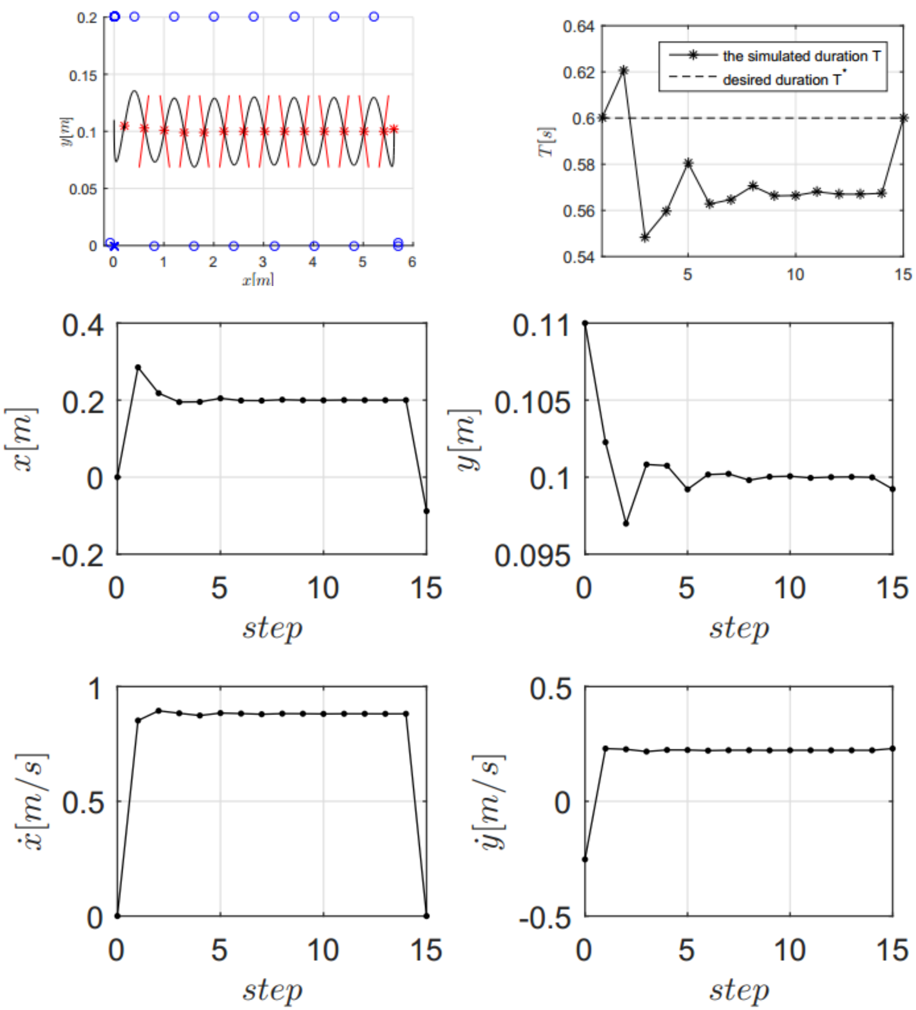
- The manifold of *C* and *T* that makes the phasing variable monotonic for the periodic gait is defined:

$$\mathcal{M} := \{(C,T)| \min_{0\leq t\leq T}\{\dot{\Phi}(C,T)\} > 0\}$$



Thus, in order to accomplish a step, the values of *C* and *T* must be located inside of the intersection of *Qsyn* and *M*

Simulations



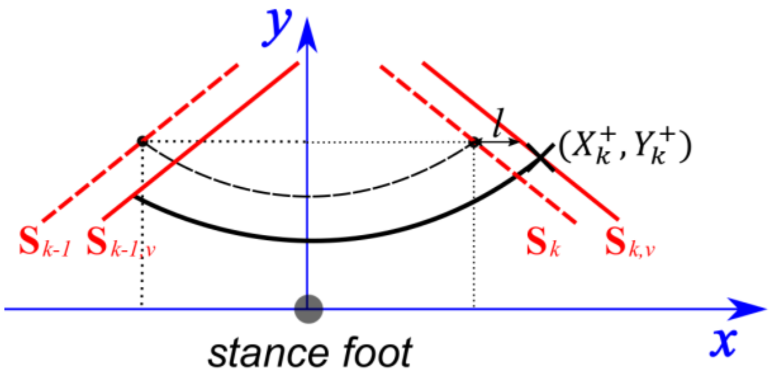
- Simulation:
  k_D=1,k_S=0,d=-0.4, C=3, T=0.6s, S=0.4m, D=0.2m

- the walking gait is self-synchronized but not self-stabilized

Condition of transition based on the position of the CoM and feedback of its velocity

- The feedback of the velocity of CoM is introduced into the condition of switching the stance leg, and a new switching manifold is proposed:

$$\mathbf{S_v} = \{(X,Y)|(X - X^{*-} - l) + C(Y - Y^{*-}) = 0\}$$



where $l = k_v(\dot{X}^{*+} - \dot{X}_k^+)$

Phasing variable for stability

- With the feedback of the velocity of the CoM along *x* axis, the position of the CoM at the end of a step is modified

$$X^{-} = X^{*-} - C\delta + l, Y^{-} = Y^{*-} + \delta$$

- Due to the condition that:

$$\Phi(X^{+}, Y^{+}) = 0$$

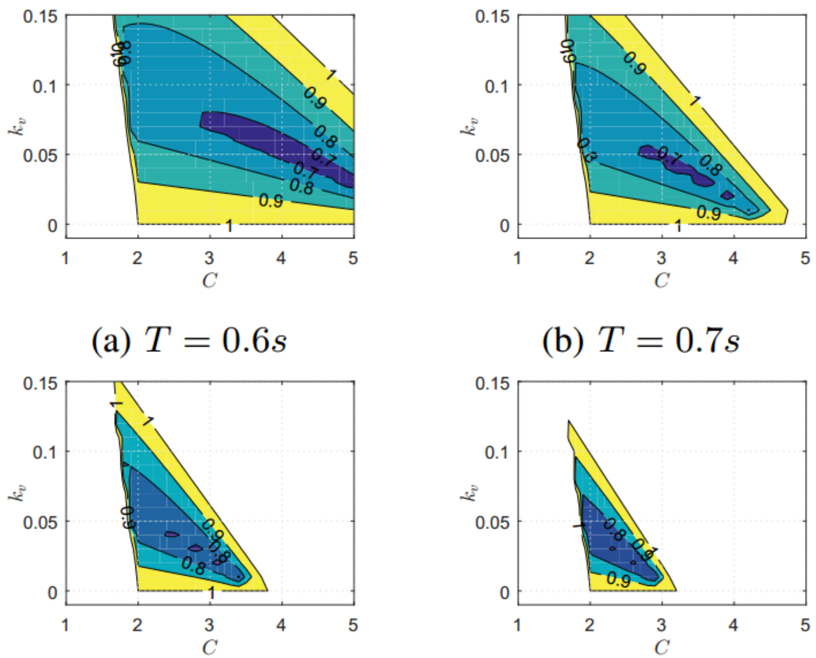
$$\Phi(X^{-}, Y^{-}) = 1$$

- ✓ The feedback of the velocity when the CoM crosses the switching surface is introduced into the function of phasing variable Φ. The new phasing variable Φ is:

$$\Phi = \frac{M_1 + dM_2M_3M_4 - dM_4l + d(M_3 - M_2)M_4l^2}{M_2 + l}$$

Stability analysis

- Using Poincaré return map to calculate the eigenvalues as function of C and k_v for different T



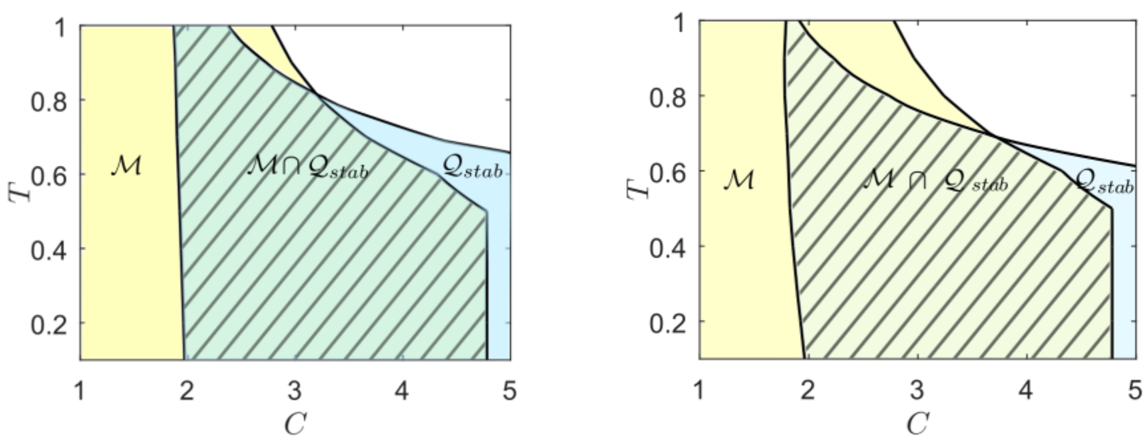
(a) $T = 0.6s$ (b) $T = 0.7s$

(c) $T = 0.8s$ (d) $T = 0.9s$

Choice of C

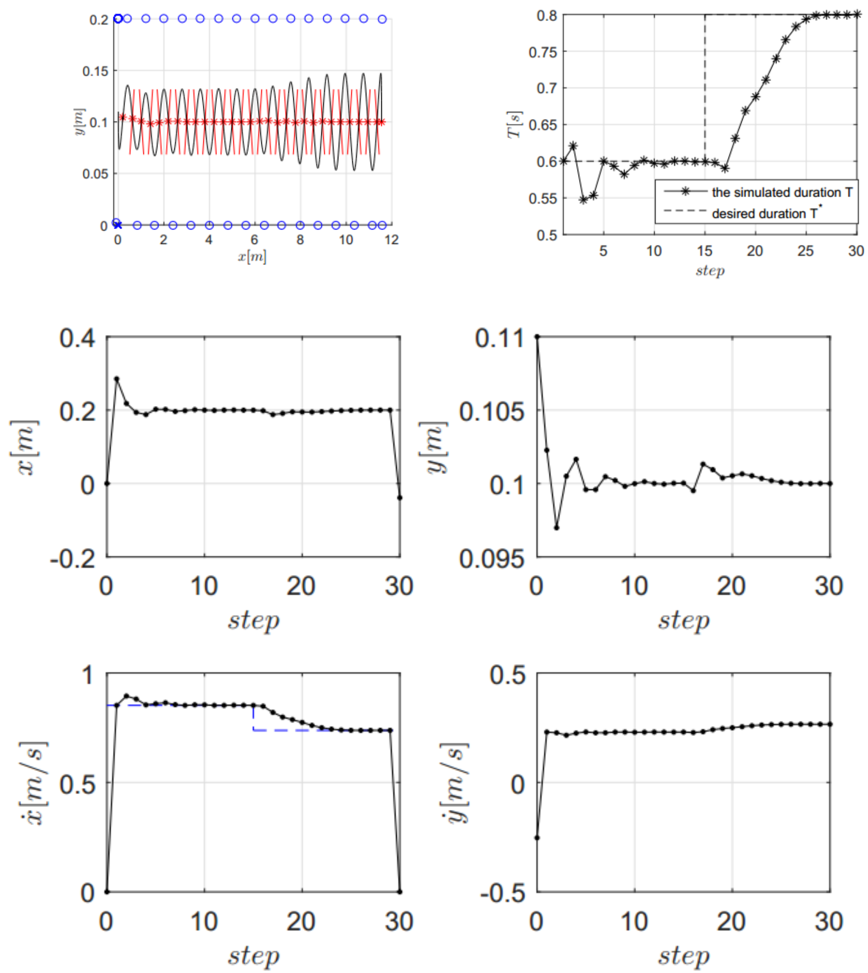
- In order to make the walking gait stable, the manifold of C and T is defined as

$$\mathcal{Q}_{stab} := \{(C,T)|\lambda_i < 1(i=1,\cdots,n)\}$$



(a) $k_v = 0.04$ (b) $k_v = 0.08$

Simulations



- Simulation:

  kD=1, kS=0, d=-0.4, C=3,

  T1=0.6s, kv=0.08;

  T2=0.8s, kv=0.04
- Self-stabilization is obtained

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Thanks

Questions?