Self-synchronization and Self-stabilization of Walking Gaits Modeled by the 3D Linear Inverted Pendulum Model

Qiuyue Luo[1], Victor De-León-Gómez[1], Anne Kalouguine[1,2], Christine Chevallereau[1], and Yannick Aoustin[1]

***Abstract*— The linear inverted pendulum (LIP) model is often used to study walking gaits, but the condition of transition from one step to the following step is often neglected while it is really important for the stability of the walk. This paper studied different landing positions of the swing foot, and different conditions to switch the stance leg, based on time or on the configuration of the robot. It is shown that self-synchronization of the motion in sagittal and frontal planes can occur or not, according to different switching conditions. Neither self-synchronization nor self-stabilization is observed when the condition of switching the stance leg is based on time or when both the step length and width are imposed. On the other hand, self-synchronization can be obtained when the condition of switching the stance leg is based on a linear combination of the position of the center of mass (CoM) along the sagittal and frontal axes. Moreover, self-stabilization can be obtained when the velocity of the CoM in the sagittal plane is taken into account.**

I. Introduction

As humanoid robots are very complex 3D systems, many simplified models have been proposed for better understanding of their dynamic behaviours. The linear inverted pendulum (LIP) model proposed by S. Kajita [3] is often used to study walking gaits. This model assumes that the vertical acceleration of the CoM is zero, consequently an analytical expression to define the CoM evolution exists and equations in sagittal and frontal planes are decoupled.

In order to generate a periodic walking gait, the condition of transition from one step to the following step has to be defined. A large number of papers assume that the stance leg exchange with a constant pace [1], [2]. However, by defining the transition as a function of time, synchronization or stabilization of the walking gait can only be achieved by applying high-level control. The notions self-synchronization and self-stabilization are used when synchronization or stabilization of walking gaits can be achieved without high-level control.

The objective of this paper is to find some physical conditions that lead to self-synchronization or self-stabilization based on a simplified model. For this we will study different conditions to switch the stance leg, based on time or on the configuration of the robot. At the same time, different landing positions of the swing foot are also studied. It is shown that according to different switching conditions, self-synchronization of the motion in sagittal and frontal planes can occur or not. This paper proves that self-synchronization is not observed when the condition of switching the stance leg is based on time or when both the step length and width are imposed, *i.e.* independent from the position of the CoM. On the other hand, self-synchronization can be obtained when the condition of switching the stance leg is based on a linear combination of the position of the CoM along the sagittal and frontal axes. Moreover, it has been proved that self-stabilization can be obtained when the feedback of the velocity of the CoM in the sagittal plane is taken into account.

[1]Qiuyue Luo, Victor De-León-Gómez, Anne Kalouguine, Christine Chevallereau and Yannick Aoustin are with Laboratoire des Sciences du Numerique de Nantes (LS2N), CNRS, Centrale Nantes, Universite de Nantes, Nantes, France, christine.chevallereau@ls2n.fr

[2]Anne Kalouguine is with Softbank Robotics

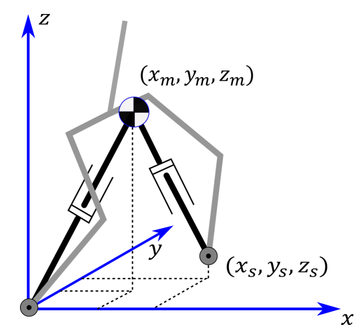

Fig. 1: A simplified model of a 3D biped robot.

II. Modeling of the walking gait via LIP model

A. Model in single support phase.

In Fig. 1, a simplified model of a 3D biped robot is illustrated. The lines connecting the two feet to the CoM are regarded as the two legs of the simplified model. The configuration of the robot is defined via the position of the CoM (x_m, y_m, z_m) with respect to the reference frame attached to the stance foot and the position of the swing foot denoted by (x_s, y_s, z_s).

For a pendulum with a constant height of CoM, the motions in the sagittal and frontal planes are decoupled. Thus, the equations of motion of the 3D LIP with respect to the reference frame attached to the stance foot are [3]:

$$\begin{aligned} \ddot{X} &= \omega^2 X \\ \ddot{Y} &= \omega^2 Y \end{aligned} \tag{1}$$

where $\omega = \sqrt{\frac{g}{z_m}}$ characterizes the LIP.

B. Hybrid model.

An overall model of walking is obtained by combining the model in single support phase and the transition model to form an hybrid system. A switching manifold is defined below:

$$\mathscr{S} := \{\mathbf{x} | z_s = 0, \dot{z}_s < 0\} \tag{2}$$

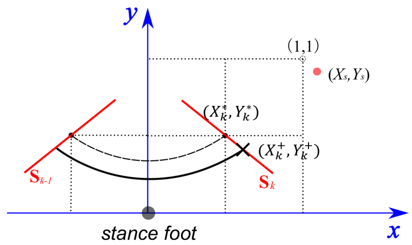

Fig. 2: The step finishes when the CoM crosses the switching manifold. The dashed line is the periodic motion.

where $\mathbf{x} := [X, Y, z_m, X_s, Y_s, z_s, \dot{X}, \dot{Y}, \dot{z}_m, \dot{X}_s, \dot{Y}_s, \dot{z}_s]^\top$ is the state of the robot. The transition model is:

$$\mathbf{x}^+ = \Delta(\mathbf{x}^-) \tag{3}$$

where Δ indicates the transition map. Thus the combination of the dynamic equations and the transition model (3) forms the single-phase hybrid dynamical system:

$$\Sigma : \begin{cases} \dot{\mathbf{x}} = f(\mathbf{x}) + g(\mathbf{x})u, & \mathbf{x}^- \notin \mathscr{S} \\ \mathbf{x}^+ = \Delta(\mathbf{x}^-), & \mathbf{x}^- \in \mathscr{S} \end{cases} \tag{4}$$

where u is the control law that allows us to track the reference trajectory of the swing foot and keep the height of the CoM to be constant.

III. The swing foot motion

A. The phasing variable

A normalized variable Φ monotonically increasing from 0 to 1 during one step, named phasing variable is defined to describe the desired trajectory of the controlled variables and to ensure the joint coordination.

IV. Condition of transition based on the position of the CoM

A. The virtual constraints

In this paper, we choose a phasing variable such that the robot switches its stance leg when the CoM crosses the switching manifold:

$$\mathbf{S} = \{(X, Y) | (X - X^{*-}) + C(Y - Y^{*-}) = 0\}. \tag{5}$$

The switching manifold $\mathbf{S}$ is defined as a line parameterized by C, represented by the red line in Fig. 2.

V. Condition of transition based on the position of the CoM and feedback of its velocity

A. The virtual constraints

The feedback of the velocity of CoM is introduced into the condition of switching the stance leg, and a new switching manifold is proposed:

$$\mathbf{S_v} = \{(X, Y) | (X - X^{*-} - l) + C(Y - Y^{*-}) = 0\} \tag{6}$$

As shown in Fig. 3, the new switching manifold is a line with an offset l from the switching condition proposed in last section. The offset is defined as $l = k_v(\dot{X}^{*+} - \dot{X}_k^+)$, where $\dot{X}_k^+$ is the velocity of the CoM along x axis at the beginning of step k, and is updated at each step.

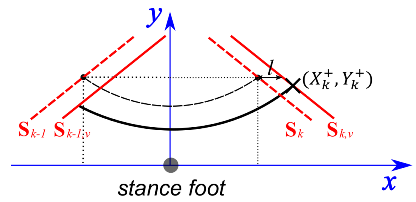

Fig. 3: The new switching condition

VI. Conclusion

In this paper, the self-synchronization and self-stabilization of a simplified model of a 3D biped robot, *i.e.* the 3D LIP model are discussed. The conditions of self-synchronization or self-stabilization of the periodic walking gait depending on the condition of transition from one step to the following step are also discussed. When the condition of transition between steps is as a function of the position of the CoM, synchronization of the walking can be "naturally" obtained. Furthermore, it has been proven that the introduction of the feedback of the velocity of the CoM into the switching condition permits to achieve self-stabilization of walking gaits.

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